Purpose: This problem set defines domain and range for functions and provides opportunities to practice in the context of piecewise functions.

Definitions: Let $f(x)$ be any function. The **domain** of f is the set of possible input values. The **range** of f is the set of possible output values.

Sometimes, the domain and range will be given to you and other times you'll have to determine a reasonable domain. Generally, we will want to express domain and range using interval notation, set builder notation, or inequalities.

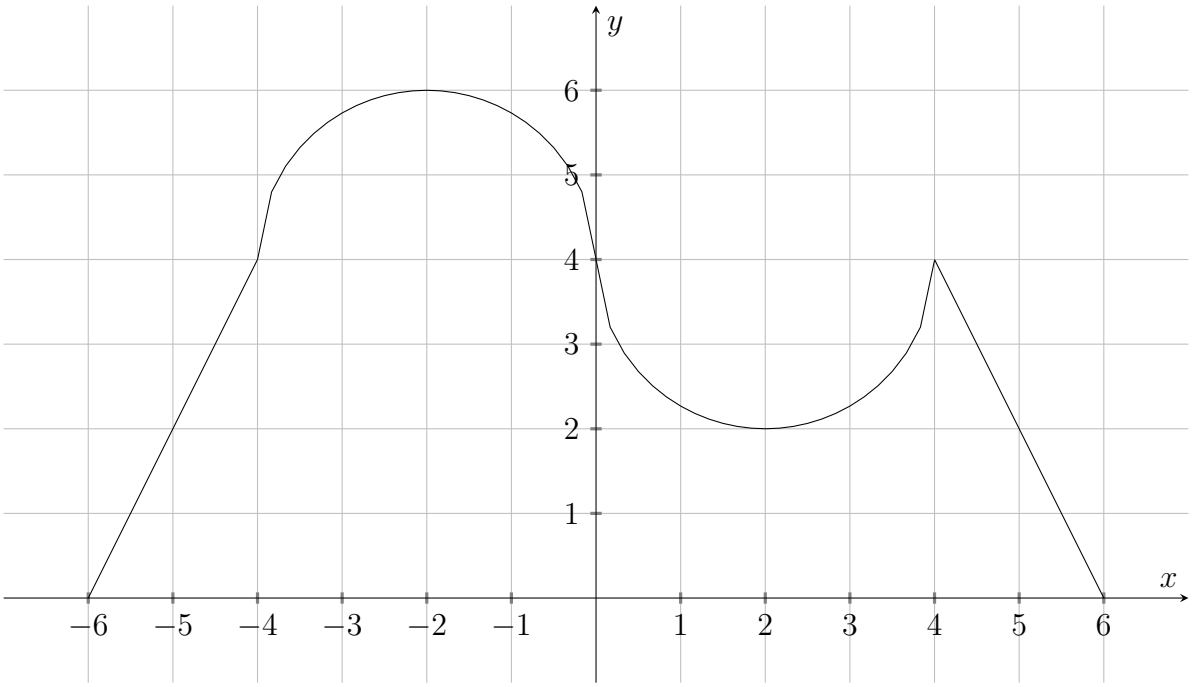
1. Write the following subset of the real numbers, $\mathbb{R}$, in interval notation, using inequalities, and in set builder notation. Also sketch the solution set on a number line.

   All numbers smaller than or equal to 2 and numbers strictly between -1 and 3.

2. The list of ordered pairs below represents a function. What is the domain of this function?

   $$\{(2, 4), (-1, 3), (0, 5), (\text{red}, 7)\}$$

3. The graph of the function $y = g(x)$ consists of line segments and semicircles of radius 2 connecting the points $(-6, 0)$, $(-4, 4)$, $(0, 4)$, $(4, 4)$, and $(6, 0)$.



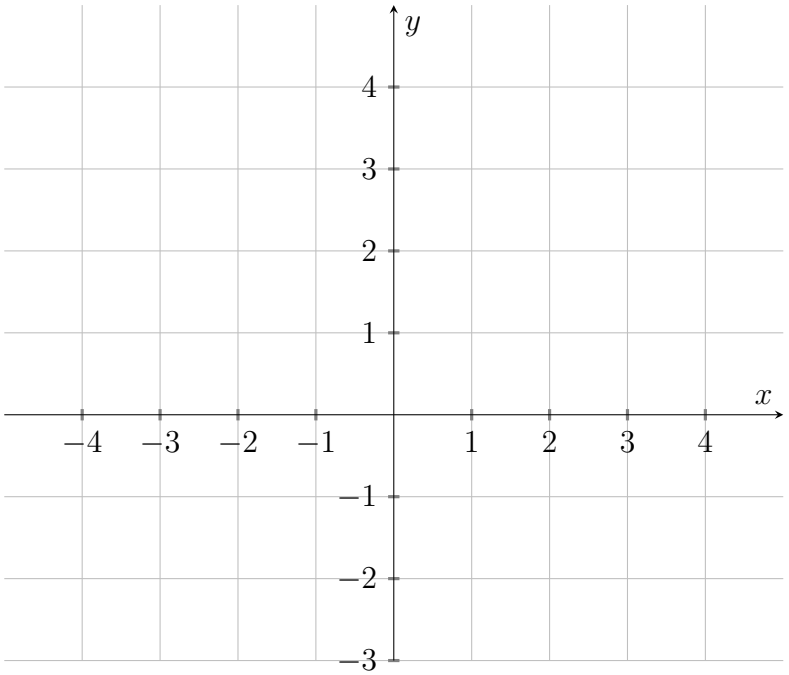
(a) What is the domain of $g(x)$? What is the range of $g(x)$?

(b) Where is the function increasing? Where is the function decreasing?

(c) If we restrict the function to the smaller domain $x \in [-5, 0]$, what is the range?

4. Sketch the graph of the function below on the coordinate axes provided on the right.

$$h(x) = \begin{cases} x+3 & \text{if } x \leq -2 \\ -1 & \text{if } -2 < x < 1 \\ x & \text{if } x \geq 1 \end{cases}.$$



5. What is the range of $h(x)$?

6. Choose any interval that is contained in the range of $h(x)$. Find, if possible, a restricted domain for $h(x)$ so that the range of $h(x)$ is the interval you chose. If it isn't possible, explain why not.

Reflection Questions:

1. Restate the definitions of domain and range.

2. Given a graph of a function, how will you determine the domain and range?

3. Given a formula for a function, what might you look for to find the domain?